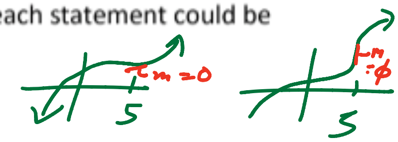
Period 3, Dec 6, 2024

4. Each of the following statements is not always true. Explain/Show why each statement could be false. Tip: One contradiction is enough.

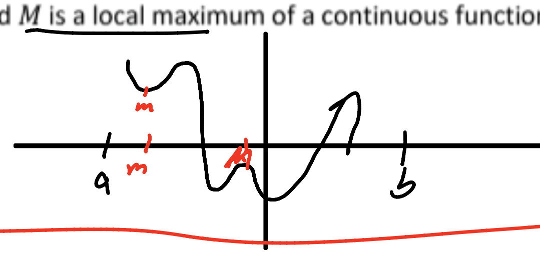
a) If $f'(5) = 0$, then there is a maximum or minimum at $x = 5$.



b) If $x = 2$ is a critical point, then $f'(2) = 0$.

c) An extrema occurs at every critical point

d) If m is a local minimum and M is a local maximum of a continuous function, then $m < M$.



a) $y = 2x^2 - 8x$ over $[0,6]$

$\frac{dy}{dx} = 4x - 8 = 0$ or $\emptyset$ $\quad \frac{d^2y}{dx^2} = 4$ concave up

$4x - 8 = 0$

$4x = 8$ $\quad$ min

$x = 2$ $\quad 2(2)^2 - 8(2) = 8 - 16 = -8$ min

c) $y = \sqrt[3]{x}$ over $[-8,8]$

$y = x^{1/3}$

$\frac{dy}{dx} = \frac{1}{3}x^{\frac{1}{3}-1} = \frac{1}{3}x^{-2/3} = \frac{1}{3\sqrt[3]{x^2}}$

0 or $\emptyset = \frac{1}{3\sqrt[3]{x^2}}$ $\quad 1 \neq 0$

$\emptyset = \frac{1}{3\sqrt[3]{x^2}}$

$x = 0$

d) $y = -x^3 + 2x^2 + 1$ over $[-1,1]$ $\quad$ Nope

$\frac{dy}{dx} = -3x^2 + 4x = 0$ or $\emptyset$

$x(-3x + 4) = 0$

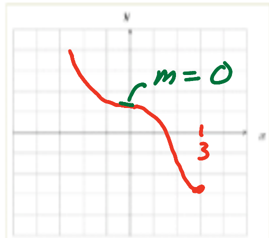
$x = 0$ or $-3x + 4 = 0$

$-4 \quad -4$

$\frac{-3x}{-3} = \frac{-4}{-3}$

$x = \frac{4}{3} = 1\frac{1}{3}$

3. Sketch a function with critical number (no extrema) at $x = 0$ and absolute minimum at $x = 3$.



6. Find the equation of the tangent line to the graph of $y^3 + (xy + 2)^2 = 0$ at the point (3, –1)

$m = \frac{2}{3}$ $(3, -1)$

$y - (-1) = \frac{2}{3}(x - 3)$

$y + 1 = \frac{2}{3}x - 2$

$-1 \quad -1$

$y = \frac{2}{3}x - 3$

$3y^2\frac{dy}{dx} + 2(xy+2)^{2-1}(1\cdot y + x\cdot\frac{dy}{dx} + 0) = 0$

$3(-1)^2\frac{dy}{dx} + 2(3(-1)+2)^1(-1 + 3\frac{dy}{dx}) = 0$

$3\frac{dy}{dx} + 2(-1)(-1 + 3\frac{dy}{dx}) = 0$ (-2)

$\frac{3dy}{dx} + 2 - 6\frac{dy}{dx} = 0$

$-3\frac{dy}{dx} = -2$; $\frac{dy}{dx} = \frac{-2}{-3} = m$

7. Solve these derivatives

a) $f(x) = e^{\cos^2 3x}$

$u = 3x$

$\frac{du}{dx} = 3$

$y = e^{\cos^2 u}$

$L = \cos u$

$\frac{dL}{du} = -\sin u$

$y = e^{L^2}$

$m = L^2$

$\frac{dm}{dL} = 2L$

$y = e^m$

$\frac{dy}{dm} = e^m$

$\frac{du}{dx} \cdot \frac{dL}{du} \cdot \frac{dm}{dL} \cdot \frac{dy}{dm}$

$3 \cdot -\sin u \cdot 2L \cdot e^m$

$-3\sin 3x \cdot 2(\cos 3x) \cdot e^{\cos^2 3x}$

$-6\sin 3x \cos 3x\, e^{\cos^2 3x} = -3(2\sin 3x \cos 3x) \cdot e^{\cos^2 3x}$

$\sin 2\theta = 2\sin\theta\cos\theta$

$3x = \theta$

$-3\sin 6x \quad e^{\cos^2 3x}$

Example 1: Find the two x-intercepts of $f(x) = x^2 - 3x + 2$ and show that $f'(x) = 0$ at some point between the two x-intercepts.

$F(x) = x^2 - 3x + 2 = (x-2)(x-1) = 0$

$x = 2 \quad x = 1$

$F(1) = 1^2 - 3(1) + 2 = 1 - 3 + 2 = 0$

$F(2) = 2^2 - 3(2) + 2 = 4 - 6 + 2 = 0$

$[1, 2]$ on the interval

$F(1) = F(2)$

$F(x)$ is continuous

$F'(x) = 2x - 3$

$F(x)$ diff

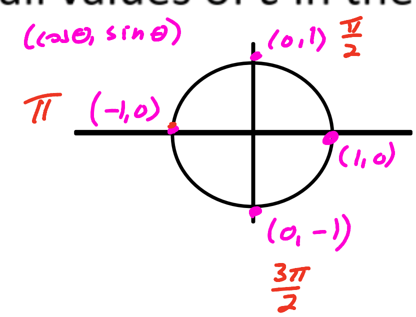
$0 = F'(x)$

$0 = 2x - 3$

$3 = 2x$

$\frac{3}{2} = x$

Example 2: Let $f(x) = \cos 2x$. Find all values of c in the interval $[-\pi, \pi]$ such that $f'(c) = 0$.



$F(-\pi) = \cos 2(-\pi) = \cos(-2\pi) = 1$

$F(\pi) = \cos 2\pi = 1$

$-\pi < c < \pi$

$F(-\pi) = F(\pi) = 1$

$F'(x) = -2\sin 2x = -2 \cdot 2 \sin x \cos x = -4 \sin x \cos x = 0$

$x = 0, \pi, 2\pi, -\pi, -2\pi$ and $x = \frac{\pi}{2}, \frac{3\pi}{2}, -\frac{\pi}{2}, -\frac{3\pi}{2}$

$c = 0, \frac{\pi}{2}, -\frac{\pi}{2}$

$a^2 - b^2 = (a-b)(a+b)$

$x^2 - 1 = x^2 - 1^2$

$4x^3 - 4x = 4x(x^2 - 1) = 4x(x-1)(x+1)$

Example 3: Given $f(x) = 5 - \frac{4}{x}$, find all values of *c* in the closed interval [1, 4] such that $f'(c) = \frac{f(4)-f(1)}{4-1}$.

$F(1) = 5 - \frac{4}{1} = 5 - 4 = 1$ (1, 1)

$F(4) = 5 - \frac{4}{4} = 5 - 1 = 4$ (4, 4)

$\frac{F(1)-F(4)}{1-4} = 1$

$F'(x) = \frac{4}{x^2} = 1$

$\frac{4}{x^2} = 1$

$x = 2$ or -2

$F(x) = 5 - 4x^{-1}$

$F'(x) = 0 - 4(-1)x^{-1-1} = 4x^{-2} = \frac{4}{x^2}$

Example 4: Verify that the function $f(t) = t^3 - 3t + 5$, for $-1 \leq t \leq 1$ satisfies the conditions for the Mean Value Theorem. Find the number(s) c.

$F(-1) = (-1)^3 - 3(-1) + 5 = -1 + 3 + 5 = 7$ (−1, 7)

$F(1) = 1^3 - 3(1) + 5 = 1 - 3 + 5 = 3$ (1, 3)

$\frac{7-3}{-1-1} = \frac{4}{-2} = -2$

$F'(T) = 3T^2 - 3$

$3T^2 - 3 = -2$

$+3 \quad +3$

$3T^2 = 1$

$T^2 = \frac{1}{3}$

$\sqrt{T^2} = \sqrt{\frac{1}{3}}$

$T = \pm \frac{1\sqrt{3}}{\sqrt{3}\cdot\sqrt{3}} = \pm \frac{\sqrt{3}}{3}$

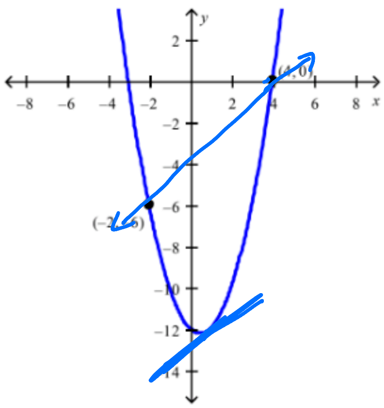
$dis = R \cdot T$

5 miles $= R \cdot 4$ min $= R \cdot \frac{1}{15}$ hours

$15 \cdot 5 = \frac{R}{15} \cdot 15$

$75 = R$
mph

Example 6



$y = x^2 - x - 12$

a) Find the equation of the secant line joining the points $(-2, -6)$ and $(4, 0)$.

$m = 1 = \frac{-6-0}{-2-4} = \frac{-6}{-6}$

$y - 0 = 1(x - 4)$

$y = x - 4$

b) Use the Mean Value Theorem to determine the point c in the interval $(-2, 4)$ such that the tangent line is parallel to the secant line.

$\frac{dy}{dx} = 2x - 1$

$1 = 2x - 1 \Rightarrow 2 = 2x \Rightarrow 1 = x$
+1 +1

$y = 1^2 - 1 - 12 = -12$

point $(1, -12)$
$m = 1$

$y - (-12) = 1(x - 1)$

$y + 12 = x - 1$
-12 -12

$y = x - 13$

For the function $f(x) = \sqrt{x}$, find the value(s) of c that satisfy the conclusion of the Mean Value Theorem on the interval $[0, 4]$.

$F(x) = x^{\frac{1}{2}}$

$F(0) = 0$

$F(4) = 2$

$\frac{2-0}{4-0} = \frac{2}{4} = \frac{1}{2}$

$\frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}} = \frac{1}{2}$

$x =$

$\frac{1}{2\sqrt{x}} = \frac{1}{2}$

$1 \cdot 2\sqrt{x} = 1 \cdot 2$

$\frac{2\sqrt{x}}{2} = \frac{2}{2}$

$\sqrt{x} = 1$

$x = 1$